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A RANDOM COEFFICIENT MODEL FOR REEXAMINING RISK DECOMPOSITION METHOD AND RISK-RETURN RELATIONSHIP TEST

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SUMMARY

A random coefficient regression is used to re-examine the traditional method of estimating beta coefficients and decomposing total risk into systematic risk and nonsystematic risk. It is found that the OLS regression method is not an appropriate method for either estimating beta coefficient or decomposing risk components.

I. Introduction

The empirical results of the capital asset pricing model (CAPM), developed by Sharpe (1964), Lintner (1965) and Mossin (1966), are generally used to investigate the diversification effect and to test the risk-return relationship. To do these empirical studies, the fixed coefficient instead of the random coefficient regression model is used to estimate the parameters of CAPM. It is well-known that the random coefficient regression model developed by Theil and Mennes (1959), Hildreth and Houck (1968) and others is a generalized case of the fixed coefficient regression model. Hence, the random coefficient regression model can be used to re-examine whether the empirical results obtained from the fixed coefficient regression model are appropriate for testing some theoretical implications of capital asset pricing.

The main purpose of this paper is to derive a random coefficient regression model for re-examining the validity of the traditional fixed coefficient regression model in estimating beta coefficients and decomposing the total risk into systematic risk and unsystematic risk. The relationship between the betas obtained from the random coefficient model and those obtained from the fixed coefficient model is investigated; implications of the traditional method in decomposing risk are also explored. In the second section, a random coefficient model is specified for estimating the parameters of CAPM. It is shown that the beta coefficients estimated from the random coefficient model is a generalized case of those obtained from the fixed coefficient model. In the third section, the validity of the traditional method of decomposing total risk is re-examined in accordance with the random coefficient model developed in the previous section; the impact of different risk decomposition methods on the

risk-return relationship test is also analyzed. In the fourth section, monthly data of 363 firms for the time period January 1966 - March 1975 are used to show how the new concepts developed in this paper can be used to decompose total risk and to perform risk-return relationship test. Finally, the results of this paper are summarized.

II. A Random Coefficient Model for Estimating Beta Coefficient and Its Population Variance

It is well-known that the random coefficient regression model developed by Theil and Mennes (1959) and Hildreth and Houck (1968) is a generalized case of the fixed regression coefficient model. The essence of the random coefficient regression model is that population variances associated with the regression coefficients can be estimated and tested. Hildreth and Houck (1968) have argued that the existence of population variances associated with the regression coefficients can be explained as the impact of omitted variables; Cooley and Prescott (1973) have argued that sequential parameter variation may arise because of problems of structural change, mis-specifications, and problems of aggregation.

The necessity of using the random coefficient model instead of the fixed coefficient model to estimate the beta coefficient can be justified by both empirical and theoretical grounds. Empirically, Cohen and Pogue (1967), Aber (1973) and Lee and Lloyd (1973) have found that the multi-index model can generally be used to improve the explanatory power of the market model. If a single-index model instead of a multi-index model is used to do empirical study, then the necessity of using the random coefficient model can be justified by the impact of omitted

variables.[1] Theoretically, Merton (1972) has shown that the investment opportunity set generally shifts over time unless the interest rate is constant over time. Furthermore, Black (1976) has argued that shocks in the capital market should be regarded as random fluctuation of the beta coefficient of a dynamic capital asset pricing model.

In accordance with either Theil and Mennes (1959) or Theil (1971, 622-627), a random coefficient CAPM for estimating beta coefficients can be defined as

$$Y_t = b_t X_t + \varepsilon_t \tag{1}$$

where

$$Y_t = R_{jt} - R_{ft},$$
$$X_t = R_{mt} - R_{ft}$$

ε_t = random disturbance with mean zero and variance σ_0^2,

R_{jt} = the rate of return on security j in time period t,

R_{mt} = the market rate of return in time period t,

R_{ft} = the risk free rate of interest,

b_t = random coefficient with mean β and variance σ_1^2.

It is assumed that ε_t is independent of b_t. Equation (1) is a linear regression model with random slope b_t. It can be rewritten into a fixed coefficent CAPM as

$$Y_t = \beta X_t + \varepsilon_t^* \tag{2}$$

where β is mean of the random coefficient b_t and $\varepsilon_t^* = (b_t - \beta)X_t + \varepsilon_t$. This is a fixed coefficient regression model with a heteroschedastic

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[1]The multi-index model is generally subject to the problem of multicollinearity. See Aber (1973).

error term; therefore, to improve efficiency, the generalized least squares method (G.L.S.) instead of the ordinary least square method (O.L.S.) should be employed to estimate b_t. Theil and Mennes (1959) have shown that equation (3) can be used to estimate both σ_0^2, the residual variance of the dependent variable Y_t, and σ_1^2, the population variance of the regression coefficient b_t, simultaneously.

$$e_t^2 = \sigma_0^2 P_t + \sigma_1^2 Q_t + f_t \tag{3}$$

where $e_t = Y_t - bX_t$, the OLS residuals,

$$\text{(a)}\quad P_t = 1 - \frac{X_t^2}{\sum_{t=1}^{n} X_t^2}$$

$$\text{(b)}\quad Q_t = X_t^2 \left[1 - 2\frac{X_t^2}{\sum_{t=1}^{n} X_t^2} + \frac{\sum_{t=1}^{n} X_t^4}{\left(\sum_{t=1}^{n} X_t^2\right)^2}\right] \tag{4}$$

and $f_t \sim N[0, 2(\sigma_0^2 P_t + \sigma_1^2 Q_t)^2]$.

To remove the difficulty of multicollinearity in estimating the variance components σ_0^2 and σ_1^2, a restricted regression is proposed to estimate the variance component.[2] The summation of equation (3) over t can be used as a constraint of the regression associated with equation (3). Therefore, a restricted regression can be defined as

$$\text{(a)}\quad \ell_\alpha^2 = \sigma_0^2 P_\alpha + \sigma_1^2 Q_\alpha + f_\alpha \qquad (\alpha = 1, 2, \ldots., n)$$

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[2] For a discussion of the multicollinearity problem associated with this kind of random coefficient model, see Lee and Chen (1976).

subject to (5)

(b) $E(\Sigma \ell_\alpha^2) = \sigma_0^2 \ \Sigma P_\alpha + \sigma_1^2 \ \Sigma Q_\alpha$

where f_α is normally distributed with mean zero and variance

$2\ (\sigma_0^2 P_\alpha + \sigma_1^2 Q_\alpha)^2$.

After imposing equation (5a) on (5b), we obtain

$$[\ell_\alpha^2 - (\Sigma \ell_\alpha^2) \frac{P_\alpha}{\Sigma P_\alpha}] = [Q_\alpha - \frac{(\Sigma Q_\alpha)\ P_\alpha}{\Sigma P_\alpha}]\ \sigma_1^2 + f_\alpha \qquad (6)$$

This is a simple regression for estimating σ_1^2. After σ_1^2 has been estimated, σ_0^2 can be estimated by the relationship defined in (5b). This method has reduced the multicollinearity problem.

As both σ_0^2 and σ_1^2 are estimated, a weighted regression model as defined in equation (7) can be used to estimate the beta coefficient.

$$Y_t W_t = \beta(X_t W_t) + \delta_t \qquad (7)$$

where

$$Y_t = R_{jt} - R_{ft}$$

$$X_t = R_{mt} - R_{ft}$$

$$W_t = (\sigma_0^2 + \sigma_1^2 X_t^2)^{-\frac{1}{2}}$$

and

$$\delta_t = \frac{\varepsilon_t^*}{(\sigma_0^2 + \sigma_1^2 X_t^2)^{\frac{1}{2}}}$$

Equation (7) is a homoscedastic regression with stochastic disturbance δ_t and its estimated slope becomes an efficient estimator.[3] In testing the heteroscedasticity of the market model, Martin and Klemkosky (1975)

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[3]See Kmenta (1971) for detail.

Rogalski and Vinson (1975), and Belkaoui (1977) do not explicitly take the heteroscedasticity associated with $\sigma_1^2 X_t^2$ into account. Therefore, their conclusions are subject to further investigation. In the following section, some theoretical implications associated with the random coefficient CAPM, developed in this section, will be investigated.

III. Risk Decomposition Method, Risk-Return Relationship And the Index of Uncertainty

The random coefficient CAPM is different from the fixed coefficient CAPM in two important aspects, i.e. (i) the population variance associated with the beta coefficient can be estimated and (ii) the weighted least squares method is used to improve the efficiency of the estimated beta coefficient. If the estimated σ_1^2 is not significantly different from zero, then the random coefficient CAPM will reduce to the fixed coefficient CAPM. Hence, the random coefficient CAPM is a generalized case of fixed coefficient CAPM.

Now, the problems associated with total risk decomposition will investigated. Following equation (1), the total risk associated with either individual security or portfolio can be decomposed as

$$\begin{aligned} Var\ (Y_t) &= Var\ (b_t\ X_{jt} + \varepsilon_t) = Var\ [\beta X_t + (b_t - \beta)\ X_t + \varepsilon_t] \\ &= (\beta^2)\ Var\ (X_t) + \sigma_1^2\ X_t^2 \\ &+ \ \sigma_0^2 \end{aligned} \tag{8}$$

If the variance associated with b_t, σ_1^2, approaches zero, then equation (8) reduces to

$$Var\ (Y_t) = \beta^2\ Var\ (X_t) + \ \sigma_0^2 \tag{9}$$

Equation (9) implies that total risk can be decomposed into systematic

risk and unsystematic risk by the OLS regression method as discussed by Francis (1976) and others. However, this result does not hold unless b_t is a deterministic variable. As the beta coefficient is stochastic, the unsystematic risk should be represented by σ_0^2. If the OLS instead of the GLS residual variance is used to perform the risk-return relationship test, then the theoretical implications can be analyzed as follows:

To investigate the relationship between the average rate of return on an individual security and its nonsystematic risk, Lintner (1965) and Douglas (1969) have employed equation (10) to do some empirical tests.

$$\bar{Y}_j = b_0 + b_1 S_j^2 + \tau_j \qquad (10)$$

where

$\bar{Y}_j$ = average rates of return for the jth security

S_j^2 = the OLS residual variance associated with the jth security.

Both b_0 and b_1 are regression parameters and τ_j is the error term associated with the second round regression.

Both Lintner and Douglas have found that $\bar{Y}_j$ is strongly correlated with S_j^2, and therefore, they argued that an individual company's average rate of return is correlated with the nonsystematic. Miller and Scholes (1972) have carefully re-examined both Lintner and Douglas's findings and failed to give a satisfactory explanation. Now, we will re-examine Lintner and Douglas's findings in accordance with the problem of risk decomposition as indicated in this section. Based upon equation (1), it can be shown that[4]

$$\bar{Y}_j = \frac{\sum_{t=1}^{n} [\beta Y_{jt} + (b_t - \beta) X_{jt} + \varepsilon_j]}{n - 1} \qquad (11)$$

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[4] For derivation of (12) see the appendix.

$$S_j^2 = \hat{\sigma}_{oj}^2 + \frac{\hat{\sigma}_{1j}^2}{n-1} \sum_{t=1}^{n} X_{jt}^2 \left[1 - \frac{\Sigma X_{jt}^4}{(\Sigma X_{jt}^2)^2}\right] \tag{12}$$

From equations (11) and (12), it is found that both $\bar{Y}_j$ and S_j^2 are functions of $(b_t - \beta) X_{jt}$ and X_t; therefore, it is not reasonable to expect that $\bar{Y}_j$ is uncorrelated with S_j^2. Furthermore, both σ_{oj}^2 and the residual variance of (7), Var (δ_t) are homoscedastic and not functions of both $(b_t - \beta) X_t$ and X_t. Therefore, both σ_{oj}^2 and Var (δ_t) will not be functions of $\bar{Y}_j$. In sum, both Lintner and Douglas's findings imply only that there exist some problems associated with decomposing the total risk into systematic risk and unsystematic risk.

The problems associated with the traditional risk-return relationship test are due to the fact that the nonsystematic risk estimate is not a true proxy. To resolve these problems, the population variance of the beta coefficient, (σ_1^2) should be estimated in accordance with the method described in the previous section. If the estimated σ_1^2 is significantly different from zero, then it implies that the beta coefficient is not stable over time. Under this circumstance, the degree of stability for the estimated beta coefficient is of interest to both security and portfolio analysts. Now, an index of uncertainty (IOU) is defined in accordance with the coefficient of variation concept as[5]

$$IOU = \hat{\sigma}_{1j} / |\hat{\beta}_j| \tag{13}$$

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[5]Kau and Lee (1977) have successfully used this kind of index to measure the degree of stability of density gradient for an urban structure study.

where

$\hat{\sigma}_{1j}$ = the estimated population standard deviation associated with the beta coefficient for the jth firm.

$\hat{\beta}_j$ = the estimated beta coefficient for the jth firm.

This index will provide a criteria for security analysts to decide whether the historical beta coefficient of a particular firm is an acceptable predictor for the future beta coefficient of that firm.

In the following section, data from 363 firms will be used to re-examine Lintner and Douglas's empirical tests on the risk-return relationship and to demonstrate how the index of uncertainty can be used to determine whether the historical beta can be used to predict the future beta.

IV. Some Empirical Results

The rates of return for 363 companies from NYSE from January 1965 through March 1975 are used to do some empirical studies in accordance with the theoretical results developed in the previous section. Both cash and stock dividends and stock splits are adjusted to obtain proper rates of return. The Standard and Poor (S & P) stock price index is employed to calculate the monthly market rate of return. The monthly treasury bill rate is used as a proxy for the risk-free rate of return. To show the different implications associated with two alternative risk decomposition methods on the risk-return relationship test, the average rates of return estimate ($\bar{R}_j$), the OLS beta coefficient estimate ($\hat{\beta}_j$), the GLS beta coefficient estimate ($\hat{\beta}_j^*$), the OLS residual variance estimate ($\hat{\sigma}_e^2$), the GLS residual variance estimate ($\hat{\sigma}_e^{*2}$), the estimated population

variance of beta coefficient ($\hat{\sigma}_i^2$) and the estimated pure regression variance ($\hat{\sigma}_0^2$) are calculated.[6] The second round regression obtained by regressing $\bar{R}_i$ on either $\hat{\beta}_i$, $\hat{\beta}_i'$, $\hat{\sigma}_e^2$, $\hat{\sigma}_e'^2$, or $\hat{\sigma}_0^2$ are calculated and the results are listed in Table I.

The result of regressing $\bar{R}_j$ on $\hat{\beta}_j$ is not significantly different from that of regressing $\bar{R}_j$ on $\hat{\beta}_j'$; however, the result of regressing $\bar{R}_j$ on $\hat{\sigma}_e^2$ is significantly different from those results obtained by regressing $\bar{R}_j$ on either $\hat{\sigma}_e'^2$ or $\hat{\sigma}_0^2$. These findings imply that the average rate of return is not significantly correlated with the unsystematic risk unless the estimated unsystematic risk is contaminated by some systematic components. In sum, the OLS regression method is not an appropriate method of decomposing total risk into systematic risk and unsystematic risk unless the population variance associated with the beta coefficient is trivial.

To compare the regression results obtained from the GLS with those obtained from the OLS, $\hat{\beta}_j$, $\hat{\beta}_j'$, the OLS coefficient of variation (R^2), the GLS coefficient of variation (R'^2) and the index of uncertainty (IOU) for the first 40 firms are listed in Table II. From the 上 statistics, it is found that GLS estimators are generally more efficient than OLS estimators.[7] The IOU as indicated in column (e) of Table II can be used to measure the degree of reliability for an estimated beta coefficient.

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[6]All estimated $\hat{\sigma}_i^2$'s are significantly different from zero and there exist only 246 estimated σ_0^2's that are positve and significantly different from zero.

[7]Since comparison of R^2 and R'^2 are not meaningful.

If the IOU's from different betas are statistically different from each other, then the comparison among different estimated beta coefficient becomes difficult. Based upon investors' subjective judgment, the IOU can be used as a criteria for determining whether historical betas are acceptable predictors of future betas or not.

V. Summary

A random coefficient regression model is proposed to re-examine the traditional method of estimating beta coefficients and decomposing total risk into systematic risk and nonsystematic risk. It is found that the OLS regression method is not an appropriate method for either estimating beta coefficients or decomposing risk components. In addition, the index of uncertainty is introduced to determine the usefulness of historical beta coefficient estimates in both security and portfolio analyses. The data of 363 firms are used to do some empirical studies in accordance with the new model developed in this paper. It is found that the empirical results generally support the theoretical conclusions derived in this study.

TABLE I

OLS and GLS Random Coefficient Regression Results

Ticker Symbols	(a) OLS $\hat{\beta}_j$	(b) GLS $\hat{\beta}_j^*$	(c) OLS R^2	(d) GLS R^2	(e) Index of Uncertainty $\sigma_1/\lvert\beta\rvert$
AMX	0.68 (5.04)	0.97 (7.67)	.181	.991	.83
AR	1.16 (9.02)	1.44 (11.81)	.313	.994	.54
N	0.84 (6.57)	0.86 (8.23)	.329	.995	.84
TC	1.10 (7.66)	1.09 (9.77)	.235	.989	.80
UC	-0.38 (-2.40)	-1.43 (-15.06)	.044	.995	.70
HDA	0.71 (4.84)	0.75 (6.35)	.197	.987	1.14
SJO	0.84 (5.74)	0.84 (6.81)	.278	.990	.87
CRK	0.24 (1.35)	0.03 (0.27)	.007	.033	31.58
DM	0.27 (1.38)	-1.17 (-11.94)	.012	.986	0.88
HM	0.12 (0.67)	-0.71 (-7.31)	.002	.965	1.44
EFU	1.35 (8.34)	1.24 (8.41)	.313	.986	.59
NC	1.18 (6.27)	1.09 (7.27)	.204	.975	.77
PCO	1.03 (7.76)	1.06 (7.30)	.234	.983	.72

Ticker Symbols	(a) OLS $\hat{\beta}_j$	(b) GLS $\hat{\beta}_j$	(c) OLS R^2	(d) GLS R^2	(e) Index of Uncertainty $\sigma_1 / \lvert\beta\rvert$
GAO	0.13 (0.64)	-0.86 (-8.58)	.004	.983	1.22
LLX	0.82 (4.67)	0.92 (7.19)	.178	.985	.93
SOC	1.01 (9.50)	1.18 (8.50)	.282	.991	.58
APC	1.14 (9.06)	1.27 (8.96)	.291	.990	.56
MDE	-0.35 (-1.88)	-0.60 (-6.20)	.019	.959	1.69
SAF	1.16 (7.92)	1.31 (8.29)	.204	.981	.60
GF	0.14 (0.64)	-0.62 (-6.18)	.009	.982	1.70
GEB	0.09 (0.50)	2.11 (21.31)	.002	.996	.49
K	0.61 (4.56)	0.61 (5.13)	.202	.986	1.30
OAT	-0.14 (-0.57)	0.82 (7.98)	.007	.985	1.31
SB	0.72 (5.62)	0.69 (6.48)	.346	.994	.96
ESM	0.03 (0.18)	0.89 (9.06)	.0003	.988	1.16
IBP	0.16 (0.87)	2.79 (28.17)	.004	.995	.37
BRY	-0.09 (-0.42)	0.48 (4.59)	.004	.969	2.29
BN	0.82 (7.81)	0.90 (7.44)	.299	.993	.76
KRA	0.76 (5.54)	0.82 (7.30)	.271	.993	.97

Ticker Symbols	(a) OLS $\hat{\beta}_j$	(b) GLS $\hat{\beta}_j$	(c) OLS R^2	(d) GLS $\hat{R}^2$	(e) Index of Uncertainty $\sigma_i/\|\beta\|$
PET	0.25 (1.44)	0.37 (3.36)	.018	.927	2.69
CPB	-0.02 (-0.10)	1.18 (11.96)	.0001	.994	.88
GG	0.08 (0.45)	0.87 (8.79)	.002	.986	1.19
LJ	0.32 (1.68)	1.00 (9.93)	.020	.986	1.06
RAL	-0.05 (-0.29)	1.12 (11.31)	.001	.993	.93
CPC	0.66 (4.68)	0.59 (5.41)	.263	.989	1.32
ACM	0.15 (0.77)	0.92 (9.35)	.013	.994	1.12
HLY	0.03 (0.12)	-0.49 (-4.75)	.0003	.957	2.21
HSY	0.68 (4.52)	0.58 (4.97)	.210	.982	1.44
WWY	0.63 (3.47)	0.48 (4.47)	.205	.980	1.84
GHB	0.85 (6.60)	0.88 (6.50)	.194	.982	.92

Remarks:

(i) t statistics in parentheses.
(ii) Ticker Symbols are used by S & P Corp.
(iii) Results of other firms are available from the authors.

TABLE II

Risk-Return Trade-off Tests

(1) OLS: $\bar{R}_j = -0.0044 - 0.0022\,\hat{\beta}_j$ $\quad$ $R^2 = .0237$
$\qquad\qquad\qquad (-6.1055) \quad (-2.9578)$

(2) GLS: $\bar{R}_j = -0.0047 - 0.0015\,\hat{\beta}_j'$ $\quad$ $R^2 = .0293$
$\qquad\qquad\qquad (-7.7021) \quad (-3.2985)$

(3) OLS: $\bar{R}_j = -0.0039 - 0.2854\,\hat{\sigma}^2_{ej}$ $\quad$ $R^2 = .0278$
$\qquad\qquad\qquad (-4.9177) \quad (-3.2075)$

(4) GLS: $\bar{R}_j = -0.0053 - 0.0899\,\hat{\sigma}'^2_{ej}$ $\quad$ $R^2 = .0077$
$\qquad\qquad\qquad (-8.0667) \quad (-1.6281)$

(5) OLS: $\bar{R}_j = -0.0062 + 0.0253\,\hat{\sigma}^2_{oj}$ $\quad$ $R^2 = .00002$
$\qquad\qquad\qquad (-14.0467) \quad (0.1040)$

APPENDIX

Let U represent the residual column vector and X represent independent variable column vector of equation (2), then the OLS residual variance in terms of σ_0^2, σ_1^2 and X_t can be derived as follows:

Following Theil (1971, 207-213), it can be shown that

$$E\left(\sum_{t=1}^{n} U_t^2\right)$$

$$= E\{U'[I - X(X'X)^{-1}X']U\}$$

$$= \text{Trace}\,[I - X(X'X)^{-1}X']\,\Omega$$

$$= \text{Trace}\,\Omega - \text{Trace}\,X(X'X)^{-1}\,\Omega$$

$$= N\hat{\sigma}_0^2 + \hat{\sigma}_1^2 \sum_{t=1}^{n} X_t^2 - \frac{\sum_{t=1}^{n} X_t^2 (\sigma_0^2 + \sigma_1^2 X_t^2)}{\sum_{t=1}^{n} X_t^2}$$

$$= (N-1)\,\hat{\sigma}_0^2 + \hat{\sigma}_1^2 \sum_{t=1}^{n} X_t^2 - \sigma_1^2 \frac{\sum_{t=1}^{n} X_t^4}{\sum_{t=1}^{n} X_t^2} \qquad \text{(A)}$$

In addition, we also know that

$$S_u^2 = E\left(\sum_{t=1}^{n} U_t^2\right) \Big/ N-1 \qquad \text{(B)}$$

Substituting (B) into (A), we obtain equation (12).

REFERENCES

1. Aber, J. W. (1973), *Beta Coefficients and Models of Security Return*. Lexington: D. C. Heath and Company.

2. Belkaoui, A. (1977), "Canadian Evidence of Heteroscedasticity in the Market Model," *Journal of Finance*, forthcoming.

3. Black, S. W. (1976), "Rational Response to Shocks in a Dynamic Model of Capital Asset Pricing," *American Economic Review*, 66, pp. 767-779.

4. Cohen, K. and J. Pogue (1967), "An Empirical Evaluation of Alternative Portfolio Selection Models," *Journal of Business*, 40, pp. 166-193.

5. Cooley, T. F. and E. C. Prescott (1973), "Systematic (Non-Random) Variation Models Varying Parameter Regression: A Theory and Some Application," *Annals of Economic and Social Measurement*, February/April, 1973, pp. 463-473.

6. Douglas, G. W. (1969), "Risk in the Equity Markets: An Empirical Appraisal of Market Efficiency," *Yale Economic Essays*, 9, p. 3

7. Francis, J. C. (1976), *Investments: Analysis and Management*, McGraw-Hill Book Co.

8. Hildreth, C. and J. P. Houck (1968), "Some Estimators for a Linear Model with Random Coefficients," *Journal of the American Statistical Association*, 63, pp. 584-595.

9. Kau, J. B. and Cheng F. Lee (1977), "A Random Coefficient Model to Estimate a Stochastic Density Gradient," *Regional Science and Urban Economics*, forthcoming.

10. Kmenta, J. (1971), *Elements of Econometrics*, The Macmillan Company, New York.

11. Lee, C. F. and S. N. Chen (1976), "Multicollinearity and Models with Random Coefficient," Submitted to *Econometrica* for Publication.

12. Lee, C. F. and W. P. Lloyd, "The Capital Asset Pricing Model Expressed as a Recursive System: An Empirical Investigation," *Journal of Financial and Quantitative Analysis*, 11, pp. 237-249.

13. Lintner, J. (1965), "Security Prices, Risk and Maximal Gains from Diversification," *Journal of Finance*, 20, pp. 587-616.

14. Martin, J. D. and R. C. Klemkosky (1975), "Evidence of Heteroscedasticity in the Market Model," *The Journal of Business*, 48, pp. 81-86.

15. Merton, R. C. (1973), "An Intertemporal Capital Asset Pricing Model," Econometrica, 41, pp. 867-87.

16. Miller, M. H. and M. Scholes (1972), "Rate of Return in Relation to Risk: A Re-examination of Some Recent Findings," in M. C. Jensen, ed., Studies in the Theory of Capital Market, New York: Praeger Publishers, 1972.

17. Mossin, J. (1966), "Equilibrium in a Capital Asset Market," Econometrica, 34, pp. 768-783.

18. Rogalski, R. and J. D. Vinso, "Heteroscedasticity and the Market Model," Columbia University Paper No. 104, 1975.

19. Sharpe, W. F. (1964), "Capital Asset Prices: A Theory of Market Equilibrium Under Condition of Risk," Journal of Finance, 19, pp. 425-442.

20. Theil, H. (1971), Principles of Econometrics, New York: John Wiley and Sons, Inc.

21. Theil, H. and L.B.M. Mennes (1959), "Conception Stochastique de Coefficients Multiplicateurs dans l'adjustement Linéarire des Séries Temporelles," Publications del l'Institut de Statistique de l'Universite de Paris, 8, pp. 221-227.